

Comment on “ Is a Circular Orbit Possible According to General Relativity?” (arXiv:1008.3553v1)

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A new paper gr-qc/1008.3553V1 by F. T. Hioe and D. Kuebel published on 20th Aug present that there is no circular orbit in general relativity [1]:

“(1) A stable circular orbit is not possible. The most “circular” orbit must have a small but nonzero eccentricity and is thus elliptical and precesses.

... ..

(3) There is no so-called innermost stable circular orbit.”

These results are very interesting and very different from the known theory in popular books and references of general relativity, for example [2].

In order to check the results of arXiv:1008.3553v1 [1], we do some direct numerical tests by using geodesic equation in general relativity

$$\ddot{x}^\mu = -\Gamma_{\alpha\beta}^\mu \dot{x}^\alpha \dot{x}^\beta. \quad (1)$$

There are two constants, one is energy E and the another angular momentum L when a test particle is along the geodesic in Schwarzschild metric.

Giving initial radius $r = 7$ and $\theta = \pi/2$, from effective potential theory, we can decide $E = 0.944911182523$ and $L = 3.5$ for a circular orbit. Here we use the unit $G = c = 1$ and the mass of black hole is 1. Numerical results with a evolution time $t = 10^4$ are shown in Fig (1). From the Figures, the orbit is circular with an acceptable error $\sim 10^{-12}$. This error is arisen from the truncation error of the calculation of E .

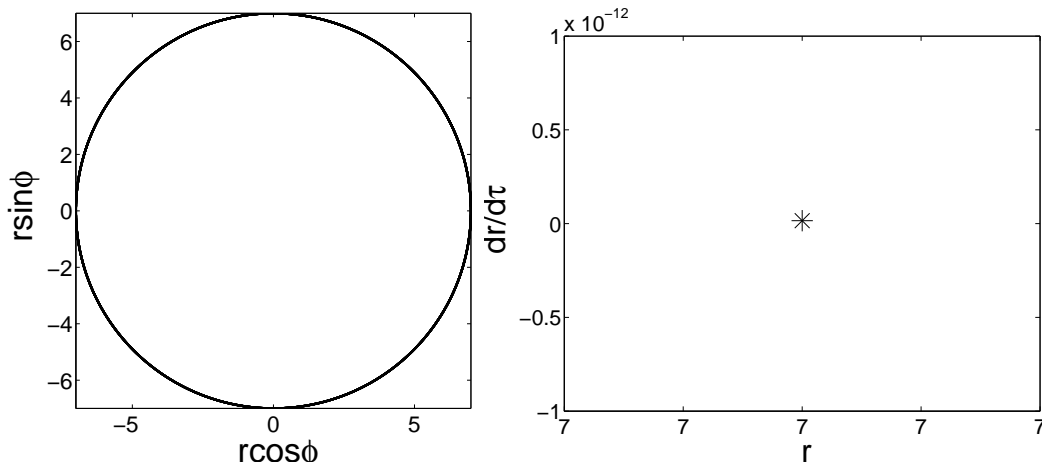


FIG. 1. Left: The orbit of a test particle with $E = 0.944911182523$, $L = 3.5$ and initial radius $r = 7$. Right: The Poincaré section of the orbit with same parameters.

From Hioe and Kuebel's paper [1], the orbit with angular momentum $L = 3.5$ must have a true eccentricity

$$\epsilon = \sqrt{2}s + \dots \approx \sqrt{2}GM/hc \approx 0.404, \quad (2)$$

this does not agree with the above numerical test. And the enough long evolution time shows that the circular orbit is stable too.

Finally, we prove the existence of ISCO ($r = 6$) in Schwarzschild spacetime. Setting $E = \sqrt{8/9}$ and $L = \sqrt{12}$, after the same evolution time, numerical code gives out a perfect stable circular orbit with zero error (double-precision), see Fig (2).

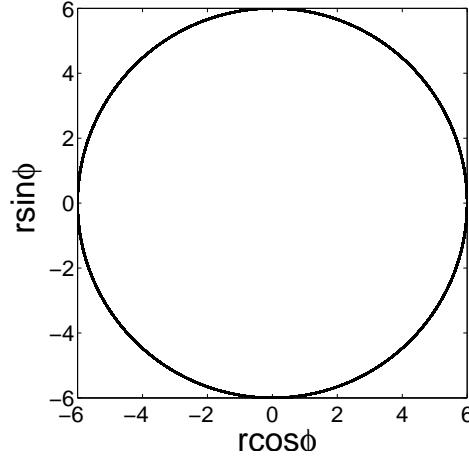


FIG. 2. The innermost stable circular orbit of a test particle around Schwarzschild black hole.

What induces the disagreement of the analysis of paper [1] and the numerical experiments? This comment thinks that the problem is the analysis of Eq.(9) and (10) in [1]. The authors defined a dimensionless parameter e and required $e \geq 0$. But actually, we do not require $e \geq 0$ even a real number! Because in Eq.(9) of [1], the key variable is $1 - e^2$. It has physical meaning once $1 - e^2$ is real. This means that e can be less than 0 or even imaginary number.

We can take the parameters of ISCO into Eq.(10) of [1],

$$e = \sqrt{\left[1 + \frac{h^2 c^2 (\kappa^2 - 1)}{(GM)^2}\right]} = \sqrt{(-1/3)} = i\sqrt{3}/3, \quad (3)$$

and $1 - e^2 = 4/3$.

In conclusion, we think that the analysis of parameter e in paper arXiv:1008.3553v1 [1] is incomplete. But the resulting question is that what is the physical explanation of the energy eccentricity parameter e defined in [1] when it is an imaginary number.

ACKNOWLEDGMENTS

Thanks a lot for the discussion with Prof. Hioe.

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- [1] F. T. Hioe, D. Kuebel, arXiv:1008.3553v1, 2010
 - [2] J. Hartle, Gravity: A introduction to Einstein's general relativity, Addison Wesley, 2002